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COMMENT

Comments on the Letter of Strnad concerning multidimensional time

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Abstract. The idea of 'reciprocity of reference frames' in the form $\boldsymbol{v}' = -\boldsymbol{v}$, which Strnad suggests should be applied generally to the relative velocities of two inertial frames, is not correct.

In a recent paper (Cole 1980), some of the transformation properties between inertial frames S and S' were presented for the case of six-dimensional relativity. In that paper, the space–time coordinates in each frame were represented by the six-vectors $x^{\mathrm{T}} = (\boldsymbol{x}^{\mathrm{T}}, \boldsymbol{t}^{\mathrm{T}})$ and $x'^{\mathrm{T}} = (\boldsymbol{x}'^{\mathrm{T}}, \boldsymbol{t}'^{\mathrm{T}})$, where superscript T denotes the transpose and $c \equiv 1$. The motions of the spatial origins 0 and 0′ of S and S' were specified in each frame in terms of their velocities and the unit tangent vectors to the projections of their space–time paths in the time subspaces. The origin 0 had unit time vectors $\boldsymbol{\alpha}_0$ in S and $\boldsymbol{\alpha}'_0$ in S', and 0′ had unit time vectors $\boldsymbol{\alpha}_{0'}$ in S and $\boldsymbol{\alpha}'_{0'}$ in S'. The velocities of 0 and 0′ in S' and S were then $\boldsymbol{v}' = \mathrm{d}\boldsymbol{x}'/\mathrm{d}t'$ and $\boldsymbol{v} = \mathrm{d}\boldsymbol{x}/\mathrm{d}t$ where $\mathrm{d}t'$ and $\mathrm{d}t$ are infinitesimal increments along the directions of $\boldsymbol{\alpha}'_0$ and $\boldsymbol{\alpha}_{0'}$ respectively. It was found that

$$|1 - v^2|^{-1/2}\boldsymbol{\alpha}_{0'} \cdot \boldsymbol{\alpha}_0 = k|1 - v'^2|^{-1/2}\boldsymbol{\alpha}'_{0'} \cdot \boldsymbol{\alpha}'_0 \tag{1}$$

where $k = +1$ and -1 for subluminal and superluminal transformations respectively, and that

$$L^{\mathrm{T}}GL = kG \tag{2}$$

where G is the 6×6 diagonal matrix with $G_{11} = G_{22} = G_{33} = -G_{44} = -G_{55} = -G_{66} = -1$.

In his letter, Strnad (1980) correctly points out that of all proposed six-dimensional schemes, this scheme is the only one viable in the sense that the standard four-dimensional theory is recovered as a special case when all time vectors are taken equal. However, he makes two main errors which lead him to wrong conclusions in the second half of his letter.

Firstly, he is not correct in his statement that invariance of $x^{\mathrm{T}}Gx$ is demanded. It is *not* demanded and, in fact, it follows from (2) that $x^{\mathrm{T}}Gx = kx'^{\mathrm{T}}Gx'$.

Secondly, he tries to prove that L is symmetric by using the idea of 'reciprocity of reference frames' in the form

$$\boldsymbol{v}' = -\boldsymbol{v} \tag{3}$$

as in the four-dimensional case for two inertial frames σ and σ' which are not

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necessarily in standard configuration. This is incorrect because, as can be deduced from many standard texts on special relativity (for example, Møller 1960, p 43), result (3) does not generally hold even in the four-dimensional case. The counter argument in the four-dimensional theory goes as follows: let σ_1 and σ_1' be inertial frames in standard configuration which are at rest relative to σ and σ' respectively. If (3) holds for the velocities $\boldsymbol{v}_1$ and $\boldsymbol{v}_1'$ of the spatial origins of σ_1' and σ_1 in σ_1 and σ_1' respectively, then we may write $\mathbf{U}\boldsymbol{v} = \boldsymbol{v}_1 = -\boldsymbol{v}_1' = -\mathbf{U}'\boldsymbol{v}'$ where $\mathbf{U}$ and $\mathbf{U}'$ are unitary matrices which in general are different since the spatial axes of σ and σ' must usually be given different rotations to bring them into standard configuration. It follows that $\boldsymbol{v}' = -\mathbf{V}\boldsymbol{v}$ where $\mathbf{V} = \mathbf{U}'^{\mathrm{T}}\mathbf{U}$ is unitary. This result must replace (3) in the general case.

From this it can be deduced that $|\boldsymbol{v}'| = |\boldsymbol{v}|$ in the four-dimensional theory. This result no longer applies in the corresponding six-dimensional theory because, in general, we do not have $\boldsymbol{\alpha}_{0'} \cdot \boldsymbol{\alpha}_0 = \boldsymbol{\alpha}'_{0'} \cdot \boldsymbol{\alpha}'_0$, so that it can be seen from (1) that v' and v are not generally equal.

Thus L is not generally symmetric, and Strnad's conclusions are not valid.

References

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